For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

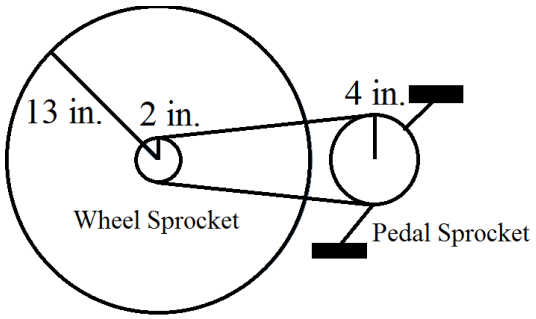
1. Consider a function $y = R(x)$ to be unattainable at a if $|R(x)| \to \infty$ as $x \to a$. Find the sum of the values of a for which $\dfrac{2x^4 - 3x^3 + x^2}{x^3 - 9x}$ is unattainable, as defined earlier in this problem.
(A) -9 (B) -6 (C) 0 (D) 6 (E) NOTA

2. Simplify the following into a trigonometric function and evaluate at $\alpha = 0$.

$$\frac{\sin(\alpha) + \sin(2\alpha) + \sin(3\alpha) + \sin(4\alpha) + \sin(5\alpha)}{\cos(\alpha) + \cos(2\alpha) + \cos(3\alpha) + \cos(4\alpha) + \cos(5\alpha)}$$

(A) $\sin(\alpha)$, 0 (B) $\cos(\alpha)$, 1 (C) $\tan(\alpha)$, 0 (D) $\sec(\alpha)$, ∞ (E) NOTA

Questions 3 and 4 refer to the following information:
Aditya is biking through the Tallahassee trails. The pedal sprocket of his bike is attached to a chain which is attached to the wheel sprocket. The pedal sprocket has a radius of 4 inches and the wheel sprocket has a radius of 2 inches. The wheel has a radius of 13 inches and turns at the same rate as its sprocket. A diagram is provided below.



3. What is the angular speed of the wheel sprocket, in radians per minute, if Aditya pedals at 30 revolutions of the pedal per minute?
(A) 30π (B) 60π (C) 120π (D) 240π (E) NOTA

4. If Aditya pedals at 30 revolutions of the pedal per minute, what is the speed of Aditya's bicycle, in inches per minute?
(A) 26π (B) 520π (C) 780π (D) 1560π (E) NOTA

5. Kyle had trouble finding the inverse of matrices. Help him manipulate the matrices below to find the value of $c_1 + c_2 + c_3$.

$$\begin{bmatrix} 3 & 6 & 2 \\ 12 & 5 & -1 \\ -2 & 2 & 12 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 53 \end{bmatrix}$$

(A) $\dfrac{16}{3}$ (B) $\dfrac{12}{5}$ (C) 6 (D) 14 (E) NOTA

6. Shardul needs to enter the Red Team's base in a game of Sky Wars. He can only enter the base and defeat his enemy if he finds the following summation. What should he answer?

$$\sum_{q=1}^{\infty} \frac{1}{q(q+1)}$$

(A) 0 (B) 1 (C) ∞ (D) Can't be evaluated (E) NOTA

7. Planet DI is an ellipse that follows the following orbital path:

$$\frac{(x-8)^2}{24} + \frac{(2-y)^2}{16} = 1$$

Assume that all units of length are in kilometers. Given that Planet DI revolves around the sun every 36 Earth-years, what is the average rate at which Planet DI sweeps out the area inside its path, relative to Earth?

(A) $\frac{2\pi\sqrt{6}}{9}$ $km^2/year$ (B) $\frac{\pi\sqrt{6}}{6}$ $km^2/year$ (C) $\frac{3\pi\sqrt{6}}{8}$ $km^2/year$ (D) $\frac{5\pi\sqrt{2}}{6}$ $km^2/year$ (E) NOTA

8. Jasmine is competing in the Balls and Urns tournament. The number of balls she has to toss into urns is equal to the number of terms in the simplified expansion of $(4b+8r-1290a+n-3d)^3$. How many balls does she have to toss?
(A) 12 (B) 15 (C) 18 (D) 35 (E) NOTA

9. Roehl is traveling through spacetime. Due to his high velocity, the axes of space and time warp such that he follows the path modeled by the equation $3x^2 - 2xy + 8y^2 - 4x + 5y + 6 = 0$. What is the shape of Roehl's path?
(A) Circle (B) Parabola (C) Ellipse (D) Hyperbola (E) NOTA

10. What is the angle of rotation needed to eliminate the xy-term in Roehl's path, modeled by the equation $3x^2 - 2xy + 8y^2 - 4x + 5y + 6 = 0$?
(A) $\frac{1}{2}\tan^{-1}\left(\frac{2}{5}\right)$ (B) $\frac{1}{2}\tan^{-1}\left(\frac{5}{2}\right)$ (C) $\tan^{-1}\left(\frac{2}{5}\right)$ (D) $\tan^{-1}\left(\frac{5}{2}\right)$ (E) NOTA

11. Meit and Rida are traveling on a plane. Their velocities can be modeled by the vectors $\langle 8, 10\rangle$ and $\langle -3, 5\rangle$ respectively. What is the angle between Meit and Rida's velocities?
(A) $\sin^{-1}\left(\frac{6}{\sqrt{1394}}\right)$ (B) $\sin^{-1}\left(\frac{13}{\sqrt{1394}}\right)$ (C) $\cos^{-1}\left(\frac{7}{\sqrt{1394}}\right)$ (D) $\cos^{-1}\left(\frac{21}{\sqrt{1394}}\right)$ (E) NOTA

12. A matrix is considered invertible if, when multiplied it by its inverse, the result is an identity matrix. Determine the product of the values of R such that the following matrix cannot be inverted.

$$\begin{bmatrix} 4 & 5 & 2 \\ 0 & 1 & 2R \\ 5 & R & 7 \end{bmatrix}$$

(A) 0 (B) $-\frac{5}{3}$ (C) $-\frac{9}{4}$ (D) $\frac{13}{8}$ (E) NOTA

13. Evaluate $(1-i)^{18}$.
(A) $256i$ (B) $-256i$ (C) $512i$ (D) $-512i$ (E) NOTA

14. The shape of Aditya's nether portal is modeled by an ellipse with equation $4x^2 + 9y^2 + 8x - 54y = -85$. Calculate the product of the latus rectum and the length of the minor axis of the portal.
(A) $\frac{64}{9}$ (B) $\frac{32}{9}$ (C) $\frac{16}{4}$ (D) $\frac{8}{3}$ (E) NOTA

15. What is the total number of distinct arrangements of the word *PRECALCUL*?
(A) 362880 (B) 181440 (C) 90720 (D) 45360 (E) NOTA

16. Stunned by the amazing creations on his coordinate plane, Aditya decides to make a vector through the origin and a point in the first quadrant. The cosine of the angle between the positive x-axis and his vector is $\frac{9}{41}$. Help Aditya find the sine of the angle between his vector and the positive x-axis.
(A) $\frac{40}{41}$ (B) $\frac{40}{9}$ (C) $\frac{9}{40}$ (D) Can't be determined (E) NOTA

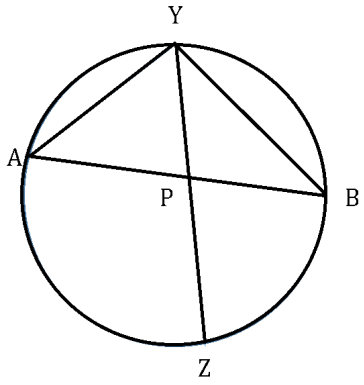
17. Wenxin and Jasmine are on a flat plain. To complete a triangle, Wenxin shoots a laser beam up towards the clouds at an angle of 45°. Jasmine, who is 570 feet away, sees the spot where the laser beam intersects a cloud, when she looks up at an angle of 75°. Assuming that Wenxin and Jasmine are both heightless, find the altitude of the cloud.
(A) $335 + 10\sqrt{2}$ (B) $350 + 36\sqrt{6}$ (C) $285 + 95\sqrt{3}$ (D) $360 + 48\sqrt{3}$ (E) NOTA

18. The eigenvalues, λ, of an $n \times n$ matrix A can be calculated by solving $\det(A - \lambda I_n) = 0$, where I_n is the identity matrix of A. Find the sum of the eigenvalues of $\begin{bmatrix} 3 & 6 \\ 9 & 1 \end{bmatrix}$.
(A) -51 (B) 0 (C) 4 (D) $\frac{1}{18}$ (E) NOTA

19. John's garden is shaped as a rhombus with side lengths of 10 feet and a larger diagonal of 12 feet. What is the larger angle between two sides of the garden?
(A) $\cos^{-1}\left(\frac{7}{25}\right)$ (B) $\cos^{-1}\left(\frac{24}{25}\right)$ (C) $\cos^{-1}\left(\frac{5}{6}\right)$ (D) $\cos^{-1}\left(\frac{6}{5}\right)$ (E) NOTA

20. Jasmine's bracelet is shaped like a circle with no clasp. If she has 6 distinct items to place on the bracelet, in how many ways can she do so?
(A) 6! (B) 5! (C) $\frac{6!}{2}$ (D) $\frac{5!}{2}$ (E) NOTA

21. Stewart has to calculate the area of a rather obscure square. Given that $AY = 5$, $AP = 3$, $AB = 8$, $YP = 6$, and $YZ = 10$, find the area of a square with side length YB using the provided diagram.



(A) $\frac{100}{3}$ (B) $\frac{283}{3}$ (C) 100 (D) 480 (E) NOTA

22. What is the sine of the reference angle of 450° multiplied by the reference angle of $\frac{9\pi}{4}$ radians?
(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{16}$ (E) NOTA

23. The derivative of a function, $f'(x)$, is defined as $f'(x) = \lim_{h\to\infty} \frac{f(x+h)-f(x)}{h}$. Find the derivative of $f(x) = 2x^2 - 3x + 5$ at $x = 12$.
(A) 0 (B) 24 (C) 45 (D) 237 (E) NOTA

24. Shardul is tossing milk cartons. There is a 70% likelihood of him making the toss on any given try. What is the probability that he makes exactly two shots out of three attempts, given each shot is independent of the other?
(A) 14.7% (B) 30.7% (C) 44.1% (D) 65.1% (E) NOTA

25. The amount of money in Azhar's piggy bank is modeled by the function $S(t) = S_0 e^{2y}$ where S_0 is the principle amount and y is the number of years that has passed. In years, when will Azhar have tripled the principle amount of money in his piggy bank?
(A) $\frac{\ln(2)}{3}$ (B) $\frac{\ln(3)}{2}$ (C) $\frac{\ln(2000)}{0.3}$ (D) $\frac{\ln(3000)}{0.2}$ (E) NOTA

26. A particle is traveling on a path modeled by the equation $p_1 = 3t + 2$ and another particle is traveling on the path modeled by the equation $p_2 = 7t + 8$. What is the smaller angle formed between the intersection of the two lines?
(A) $\tan^{-1}\left(\frac{21}{4}\right)$ (B) $\tan^{-1}\left(\frac{4}{21}\right)$ (C) $\tan^{-1}\left(\frac{11}{2}\right)$ (D) $\tan^{-1}\left(\frac{2}{11}\right)$ (E) NOTA

27. Reed's speed is equal to the characteristic of the logarithm $\log_3 6969$. What is the product of the digits of Reed's speed, assuming there are no leading zeros?
(A) 2 (B) 12 (C) 16 (D) 25 (E) NOTA

28. On the quest for freedom from Quizlets, Aditya has to determine the shape of an organic molecule! He sees that the molecule can be graphed on a polar coordinate system with equation $r = 5 + 3\cos(\theta)$. What type of polar curve is the molecule?
(A) Lemniscate (B) Cardiod (C) Rose (D) Spiral (E) NOTA

29. Jasmine is applying for the Cool Asian Squad. To get in she must find the sum of the values of θ over $[0, 2\pi]$ that satesfies: $|\sin(2\theta)| = \frac{1}{2}$. What should she say to get into that exclusive club?
(A) 2π (B) 4π (C) 8π (D) 16π (E) NOTA

30. Given that $\sin(\theta) - \cos(\theta) = \frac{3}{4}$, find $\sin(2\theta)$.
(A) $\frac{3}{2}$ (B) $\frac{7}{16}$ (C) $\frac{7}{32}$ (D) Can't be determined (E) NOTA